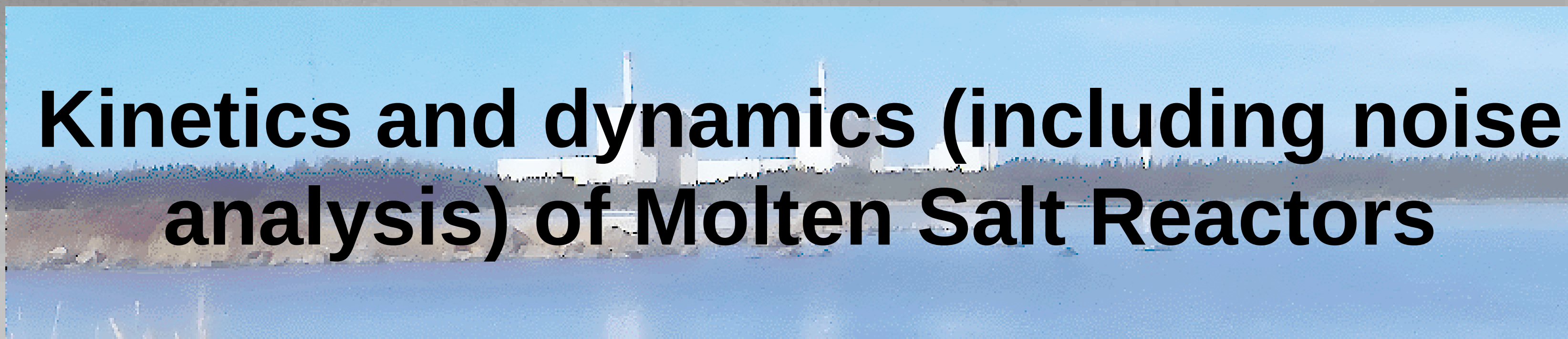


Kinetics and dynamics (including noise analysis) of Molten Salt Reactors

Imre Pázsit

Chalmers University of Technology

Division of Subatomic and Plasma Physics

MSR Summer School

2 – 4 July 2017 • Lecco, Lake Como, Italy

Objectives

- Molten salt systems (MSR) have both static and dynamic properties different from those of traditional reactors
- Objective of this lecture: to show the new and interesting physics that the MSR systems exhibit, through investigating the statics, kinetics, dynamics and neutron noise diagnostics of such systems
- Solutions in simple models give insight into the physics/neutronics of MSRs.

Objectives (cont)

- To this order, closed form analytical solutions are derived for both the static and the dynamic equations.
- The dynamic transfer properties of MSR are investigated
- The results for the dynamic case show the effect of stronger neutronic coupling and more spatially global response to localised pertubations
- At the same time the kinetic approximations become more complicated, and some intriguing theoretical questions arise.

Contents

1. Definition of the model used. Static and time-dependent equations
2. Discussion of the non-adjoint property of the static MSR equations. Construction of the adjoint
3. Interpretation of the various terms of the integro-differential form of the static equation. Some limiting cases and corresponding simplified models
4. The dynamic equations in the frequency domain: small fluctuations (neutron noise). A primer in power reactor noise.
5. System properties: the kinetic transfer function (Green's) function in various MSR models

Contents (continued)

6. The point kinetic approximation and the point kinetic component
7. The neutron noise in an MSR, induced by propagating perturbations

The material of this lecture is largely collected from Chapter 5 of the newly published book

"Molten Salt Reactors and Thorium Energy", Ed. Thomas Dolan, Woodhead Publishing Series in Energy, Elsevier, 2017

WOODHEAD PUBLISHING SERIES IN ENERGY

Molten Salt Reactors is a comprehensive reference on the status of molten salt reactor (MSR) research and thorium fuel utilization.

There is growing awareness that nuclear energy is needed to complement intermittent energy sources and to avoid pollution from fossil fuels. Light water reactors are complex, expensive, and vulnerable to core melt, steam explosions, and hydrogen explosions, so better technology is needed. MSRs could operate safely at nearly atmospheric pressure and high temperature, yielding efficient electrical power generation, desalination, actinide incineration, hydrogen production, and other industrial heat applications.

Coverage includes:

- Motivation -- why are we interested?
- Technical issues – reactor physics, thermal hydraulics, materials, environment, ...
- Generic designs -- thermal, fast, solid fuel, liquid fuel, ...
- Specific designs – aimed at electrical power, actinide incineration, thorium utilization, ...
- Worldwide activities in 23 countries
- Conclusions

This book is a collaboration of 58 authors from 23 countries, written in cooperation with the International Thorium Molten Salt Forum. It can serve as a reference for engineers and scientists, and it can be used as a textbook for graduate students and advanced undergrads.

Molten Salt Reactors is the only complete review of the technology currently available, making this an essential text for anyone reviewing the use of MSRs and thorium fuel, including students, nuclear researchers, industrial engineers, and policy makers.

Professor Dolan has worked on nuclear technology and international relations issues for three universities, five national laboratories, and in China, India, Japan, Korea, and Russia. He has worked in industry (Phillips Petroleum) and served as Physics Section Head at the International Atomic Energy Agency in Vienna, where he facilitated international cooperation on research reactors, low energy accelerators, nuclear instrumentation, and nuclear fusion research, including organization of the semi-annual IAEA Fusion Energy Conferences. He has published textbooks on Fusion Research (Pergamon 1982) and Magnetic Fusion Technology (Springer 2013).

Cover picture: Generic molten salt reactor diagram, showing graphite core (upper left), fuel tubes (yellow), drain tanks (lower left), intermediate salt loop (center), and energy conversion system (right). From Alvin Weinberg, Oak Ridge National Laboratory, 2004.

ELSEVIER WP WOODHEAD PUBLISHING An imprint of Elsevier • elsevier.com

ISBN 978-0-08-101126-3

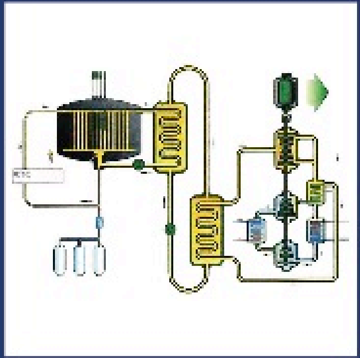
9 780081 011263

Molten Salt Reactors and Thorium Energy

Dolan

WP WOODHEAD PUBLISHING

WOODHEAD PUBLISHING SERIES IN ENERGY

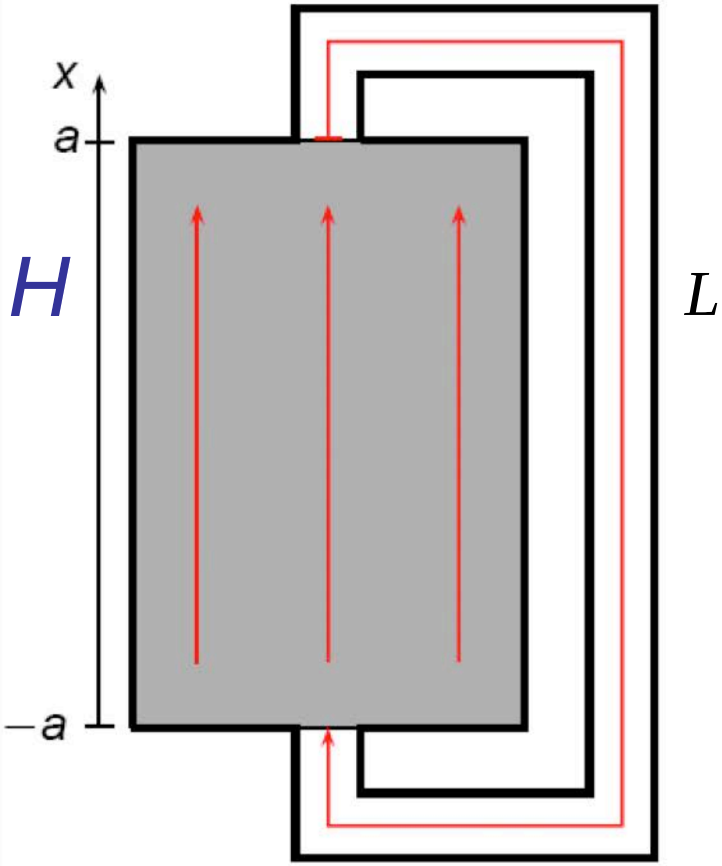


Molten Salt Reactors and Thorium Energy

Edited by Thomas J. Dolan

WP WOODHEAD PUBLISHING

1. A simple MSR model:
1- D 1- group, 1 delayed neutron group



- One-dimensional core, dimension $H = 2a$, time of passage $\tau_H = H/u$
- Extra-core piping, length L, time of passage $\tau_L = L/u$

Some definitions

Fuel velocity = *u*

Core height: *H* $\tau_c = \frac{H}{u}$ core transit time

External loop: *L*; $\tau_l = \frac{L}{u}$ loop transit time

Total length: *T* = *H* + *L*; $\tau = \frac{T}{u}$ total tr. time

Static equations

$$D\nabla^2\phi_0(z) + \left[\nu\Sigma_f(1-\beta)-\Sigma_a\right]\phi_0(z) + \lambda C_0(z) = 0 \quad (1)$$

$$u\frac{\partial C_0(z)}{\partial z} + \lambda C_0(z) - \beta\nu\Sigma_f\phi_0(z) = 0 \quad (2)$$

Boundary conditions:

$$\phi_o(z=0) = \phi_0(z=H) = 0 \quad (3)$$

$$C(0) = C(H)e^{-\lambda\tau_l} \quad (4)$$

Delayed neutron precursors do not disappear from the static equations.

Static equations

Eqs (1) and (2) can also be written in a matrix form as

$$\begin{bmatrix} D\nabla^2 + [\nu\Sigma_f(1-\beta) - \Sigma_a] & \lambda \\ -\beta\nu\Sigma_f & u\cdot\nabla + \lambda \end{bmatrix} \begin{bmatrix} \phi(z) \\ C(z) \end{bmatrix} \equiv$$

$$\equiv \mathbf{M} \begin{bmatrix} \phi(z) \\ C(z) \end{bmatrix} = 0 \qquad (5)$$

where the matrix M is defined by the first row.

Time dependent equations

$$\frac{1}{v}\frac{\partial \phi(z,t)}{\partial t} = D\nabla^2\phi(z,t) + \left[\nu\Sigma_f(1-\beta) - \Sigma_a(z,t)\right]\phi(z,t) + \lambda C(z,t) \quad (6)$$

$$\frac{\partial C(z,t)}{\partial t} + u\frac{\partial C(z,t)}{\partial z} = \beta\nu\Sigma_f\phi(z,t) - \lambda C(z,t) \quad (7)$$

Boundary conditions:

$$\phi(z=0,t) = \phi_0(z=H,t) = 0 \quad (8)$$

$$C(0,t) = C(H,t-\tau_l)e^{-\lambda\tau_l} \quad (9)$$

This latter equation will make it difficult to define a dynamic adjoint function (see later)

2. The non-adjoint property of the static equations

The MSR equations are not self-adjoint even in *1-group diffusion theory:*

$$\begin{bmatrix} D\nabla^2 + [\nu\Sigma_f(1-\beta) - \Sigma_a] & \lambda \\ -\beta\nu\Sigma_f & u\cdot\nabla + \lambda \end{bmatrix} \begin{bmatrix} \phi(z) \\ C(z) \end{bmatrix} \equiv \mathbf{M} \begin{bmatrix} \phi(z) \\ C(z) \end{bmatrix} = 0 \quad (10)$$

Then, for arbitrary functions $\phi, C, \phi^\dagger, C^\dagger$, where C and $C^\dagger$ fulfil the same boundary conditions, one has

$$\left\langle [\phi^\dagger, C^\dagger]\mathbf{M} \begin{bmatrix} \phi(z) \\ C\ (z) \end{bmatrix} \right\rangle \neq \left\langle [\phi, C]\mathbf{M} \begin{bmatrix} \phi(z)^\dagger \\ C\ (z)^\dagger \end{bmatrix} \right\rangle \quad (11)$$

where the $\langle \cdots \rangle$ sign stands for integration over the reactor volume.

The non-adjoint property of the static equations

For being self-adjoint, one should have

L.H.S. – R.H.S =0

The M_{11} term fulfils this condition. However, in general

$$\left\langle C^{\dagger} M_{22}\, C \right\rangle - \left\langle C\; M_{22}\, C^{\dagger} \right\rangle = u \int_0^H \left\{ C^{\dagger}(z) \frac{dC(z)}{dz} - C(z) \frac{dC^{\dagger}(z)}{dz} \right\} \neq 0$$

To have this term to disappear, similarly to the angle- and/or energy dependent transport equation, one needs to define an adjoint operator, and different boundary conditions for the adjoint precursors.

Construction of the adjoint operator and adjoint functions

$$\begin{bmatrix} D\nabla^2 + [\nu\Sigma_f(1-\beta) - \Sigma_a] & \lambda \\ -\beta\nu\Sigma_f & u\cdot\nabla + \lambda \end{bmatrix} \begin{bmatrix} \phi(z) \\ C(z) \end{bmatrix} \equiv \mathbf{M} \begin{bmatrix} \phi(z) \\ C(z) \end{bmatrix} = 0 \quad (12)$$

$$\begin{bmatrix} D\nabla^2 + [\nu\Sigma_f(1-\beta) - \Sigma_a] & -\beta\nu\Sigma_f \\ \lambda & -u\cdot\nabla + \lambda \end{bmatrix} \begin{bmatrix} \phi^\dagger(z) \\ C^\dagger(z) \end{bmatrix} \equiv \mathbf{M}^\dagger \begin{bmatrix} \phi^\dagger(z) \\ C^\dagger(z) \end{bmatrix} = 0 \quad (13)$$

Boundary conditions:

$$\phi^\dagger(z=0) = \phi^\dagger(z=H) = 0 \quad (14)$$

$$C^\dagger(0) = C^\dagger(H)e^{+\lambda\frac{L}{u}} = C^\dagger(H)e^{+\lambda\tau_l} \quad (15)$$

Proof of adjointness

$$\left\langle [\phi^\dagger, C^\dagger]\mathbf{M}\begin{bmatrix}\phi(z)\\ C(z)\end{bmatrix}\right\rangle = \left\langle [\phi, C]\mathbf{M}\begin{bmatrix}\phi(z)^\dagger\\ C(z)^\dagger\end{bmatrix}\right\rangle \qquad (16)$$

L.H.S. – R.H.S. =

$$u\int_0^H \left\{C(z)\frac{dC^\dagger(z)}{dz} + C^\dagger(z)\frac{dC(z)}{dz}\right\} = u\Big[C(z)C^\dagger(z)\Big]_{z=o}^{z=H} =$$

$$u\Big[C(H)C^\dagger(H) - C(0)C^\dagger(0)\Big] = \qquad (17)$$

$$u\Big[C(H)C^\dagger(H) - C(H)e^{-\lambda\tau_l}C^\dagger(H)e^{+\lambda\tau_l}\Big] = 0$$

Remark

There is one important difference compared to the traditional transport equation. There, the adjoint boundary conditions are formulated (for two opposite directions than those for the direct flux) *at the same space point at the same time*.

This is not valid for the MSR case. From (9), (15) and (17) it is seen that they express a relationship at *different points at different times*. This makes the definition of the adjoint function in the time-dependent case impossible.

3. Interpretation of the static equation

Eliminating the precursors by quadrature, one obtains the integro-differential equation

$$\nabla^2 \phi_0(z) + B_0^2 \phi_0(z) +$$

$$+e^{-z\frac{\lambda}{u}} \frac{\lambda \beta \nu \Sigma_f}{Du} \left(\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}} \int_0^H e^{z'\frac{\lambda}{u}} \phi_0(z')dz' + \int_0^z e^{z'\frac{\lambda}{u}} \phi_0(z')dz' \right) = 0$$

$$B_0^2 = \frac{\nu\Sigma_f(1-\beta) - \Sigma_a}{D} < \frac{\pi}{2a}$$

Note that only the full recirculation time τ appears in the equation.

Physical meaning of the integral terms

$$\nabla^2\phi_0(z) + B_0^2\phi_0(z) +$$

$$+\lambda e^{-z\frac{\lambda}{u}} \frac{\beta\nu\Sigma_f}{Du}\left(\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}\int_0^H e^{z'\frac{\lambda}{u}}\phi_0(z')dz' + \int_0^z e^{z'\frac{\lambda}{u}}\phi_0(z')dz'\right) = 0$$

$$C_0(z) = \frac{\beta\nu\Sigma_f}{u}\left(\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}\int_0^H e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz' + \int_0^z e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz'\right)$$

Traditional reactor: $C_0(z) = \beta\nu\Sigma_f\int_{-\infty}^t e^{-\lambda(t-t')}\phi_0(z)dt'$

Comparison with the traditional case

$$\frac{\partial C(z,t)}{\partial t} = \beta\nu\Sigma_f \phi(z,t) - \lambda C(z,t)$$

$$C(z,t) = \beta\nu\Sigma_f \int_{-\infty}^{t} e^{-\lambda(t-t')}\phi(z,t)dt'$$

In the stationary (time-independent) case:

$$C_0(z) = \beta\nu\Sigma_f \int_{-\infty}^{t} e^{-\lambda(t-t')}\phi_0(z)dt'$$

Moving precursors: infinite reactor

Neutrons generated at time $t' < t$ were born at

$$z' = z - u(t - t')$$

Hence, substituting

$$dt' = dz'/u; \qquad t - t' = \frac{z - z'}{u}$$

$$C_0(z) = \beta\nu\Sigma_f \int_{-\infty}^{t} e^{-\lambda(t-t')}\phi_0(z)dt' \Rightarrow$$

$$C_0(z) = \frac{\beta\nu\Sigma_f}{u}\int_{-\infty}^{z} e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz'$$

Moving precursors in a finite reactor 0≤ z ≤ H

Taking into account that the precursors do not move on an infinite long line, rather they recirculate, and they are only generated in the core between $0 \leq z \leq H$, we need to break up the infinite integral to sums of finite integrals with the corresponding time delays:

$$C_0(z) = \frac{\beta\nu\Sigma_f}{u}\left(\sum_{n=1}^{\infty}\int_0^H e^{-\frac{\lambda}{u}(z-z')-n\lambda\tau}\phi_0(z')dz' + \int_0^z e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz'\right)$$

The different terms in the sum correspond to the once, twice, three times recirculated precursors

Moving precursors in a finite reactor (cont)

But this is the same as what we get from the MSR equation

$$C_0(z) = \frac{\beta\nu\Sigma_f}{u}\left(\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}\int_0^H e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz' + \int_0^z e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz'\right)$$

if we use the Taylor expansion:

$$\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}} = e^{-\lambda\tau} + e^{-2\lambda\tau} + e^{-3\lambda\tau}\ldots = \sum_{n=1}^{\infty} e^{-n\lambda\tau}$$

$$C_0(z) = \frac{\beta\nu\Sigma_f}{u}\left(\sum_{n=1}^{\infty}\int_0^H e^{-\frac{\lambda}{u}(z-z')-n\lambda\tau}\phi_0(z')dz' + \int_0^z e^{-\frac{\lambda}{u}(z-z')}\phi_0(z')dz'\right)$$

Simplification 1: no recirculation

$$\nabla^2\phi_0(z) + B_0^2\phi_0(z) +$$

$$+e^{-z\frac{\lambda}{u}}\frac{\lambda\beta\nu\Sigma_f}{Du}\left(\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}\int_0^H e^{z'\frac{\lambda}{u}}\phi_0(z')dz' + \int_0^z e^{z'\frac{\lambda}{u}}\phi_0(z')dz'\right) = 0$$

For L = ∞ $(\tau = \infty)$: the first term can be neglected

$$\nabla^2\phi_0(z) + B_0^2\phi_0(z) + \frac{\lambda\beta\nu\Sigma_f}{Du}\int_0^z e^{-(z-z')\frac{\lambda}{u}}\phi_0(z')dz' = 0$$

Does not lead to much simplifications. Good for some conceptual studies.

Simplification 2: infinite fuel speed (full recirc.)

$$\nabla^2\phi_0(z) + B_0^2\phi_0(z) +$$

$$+e^{-z\frac{\lambda}{u}}\frac{\lambda\beta\nu\Sigma_f}{Du}\left(\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}\int_0^H e^{z'\frac{\lambda}{u}}\phi_0(z')dz' + \int_0^z e^{z'\frac{\lambda}{u}}\phi_0(z')dz'\right) = 0$$

For $u = \infty \; (\tau = 0)$: the second term can be neglected

$$\nabla^2\phi_0(z) + B_0^2\phi_0(z) + \frac{\beta\nu\Sigma_f}{DT}\int_0^H \phi(z')\,dz' = 0$$

Analytical solutions exist for both the static and the dynamic problem.
These equations are **also self-adjoint**.

Justification of infinite velocity (Sandra Dulla): criticality, as a function of circulation speed:

Case	u [cm/s]	τ_ℓ [s]	k_{eff}	$\Delta\rho$ [pcm]
(a)	0	$\rightarrow \infty$	1.00000	0
(b)	0.1	1000	0.99997	-2
(c)	1	100	0.99868	-132
(d)	2	50	0.99736	-265
(e)	5	20	0.99587	-415
(f)	10	10	0.99543	-459
(g)	50	2	0.99526	-476
(h)	$\rightarrow \infty$	$\rightarrow 0$	0.99526	-476

Static solution with infinite fuel speed

$$\nabla^2\phi_0(x) + B_0^2\phi_0(x) + \frac{\eta_0}{T}\int_{-a}^{a}\phi_0(x')dx' = 0$$

$$B_0^2 = \frac{\nu\Sigma_f(1-\beta) - \Sigma_a}{D} < \left(\frac{\pi}{2a}\right)^2; \quad \eta_0 = \frac{\nu\Sigma_f\beta}{D}$$

Solution:

$$\phi_0(x) = A[\cos B_0 x - \cos B_0 a]; \qquad C_0 = A\frac{B_0^2 D}{\lambda}\cos(B_0 a) = const$$

Criticality equation

$$B_0^2\cos B_0 a + \frac{2a\eta_0}{T}\cos B_0 a - \frac{2\eta_0}{TB_0}\sin B_0 a = 0$$

Full solution

The full integro-differential equation has a compact analytic solution, which can be seen if it is converted into a pure differential equation:

$$\phi_0'''(z) + \frac{\lambda}{u}\phi_0''(z) + B_0^2\phi_0'(z) + \frac{\lambda}{u}(B_0^2 + \frac{\beta\nu\Sigma_f}{D})\phi_0(z) = 0$$

Characteristic equation:

$$k^3 + \frac{\lambda}{u}k^2 + B_0^2 k + \frac{\lambda}{u}(B_0^2 + \frac{\beta\nu\Sigma_f}{D}) = 0$$

On physical grounds we expect

$$k_{1,2} = \alpha \pm i\beta; \qquad k_3 = \gamma$$

Solution (cont)

$$\phi_0(z) = A_1 e^{\alpha z}\sin(\beta z) + A_2 e^{\alpha z}\cos(\beta z) + A_3 e^{\gamma z}$$

Two coefficients can be eliminated by the boundary conditions:

$$\phi_0(z) = A[e^{\alpha z}\sin\beta z(e^{\gamma H} - e^{\alpha H}\cos\beta H) - e^{\alpha H}\sin\beta H(e^{\gamma z} - e^{\alpha z}\cos\beta z)]$$

Or, in the x-coordinate system, in the reactor centre:

$$\phi_0(x) = A\{e^{\alpha x}\cos(\beta x) - e^{-(\alpha-\gamma)a}\cos(\beta a)e^{\gamma x} - -\cot(\beta a)\tanh[(\alpha-\gamma)a][e^{\alpha x}\sin(\beta x) + e^{-(a-\gamma)a}\sin(\beta a)e^{\gamma x}]\}$$

Criticality condition

Substituting the solution back into the original equation gives the criticality condition. This can be written symbolically as

$$0 = \sum_{n=1}^{3} \frac{A_n}{\alpha_n + \lambda / u} \left(-1 + \frac{1}{e^{\lambda\tau} - 1} \left(e^{(\alpha_n + \lambda/u)H} - 1 \right) \right)$$

In reality this is much more complicated, because the relationship between the A_n has to be used explicitly.

Reverting to the case of infinite velocity

$$k^3 + \frac{\lambda}{u}k^2 + B_0^2 k + \frac{\lambda}{u}(B_0^2 + \frac{\beta\nu\Sigma_f}{D}) = 0$$

$$\Rightarrow \; k^3 + B_0^2 k = 0; \qquad \alpha = \gamma = 0; \quad \beta = B_0$$

Then the full solution will revert to that obtained before

$$\phi_0(x) = A\{e^{\alpha x}\cos(\beta x) - e^{-(\alpha-\gamma)a}\cos(\beta a)e^{\gamma x} -$$
$$-\cot(\beta a)\tanh[(\alpha-\gamma)a][e^{\alpha x}\sin(\beta x) + e^{-(a-\gamma)a}\sin(\beta a)e^{\gamma x}]\}$$

$$\Rightarrow \quad \phi_0(x) = A(\cos B_0 x - \cos B_0 a)$$

4. Neutron fluctuations in an MSR

- Why would one be interested in neutron fluctuations and neutron noise in an MSR?
- Because neutron noise diagnostics has proved to be very effective for surveillance of the operation of the existing reactors:

  - early discovery of anomalies
  - measuring operational parameters in a non-intrusive way
- There are reasons to believe that the same methods would be just as useful in an MSR

Neutron fluctuations in a power reactor

- Technological processes in the core (vibrations of control rods, boiling of the coolant in a BWR etc) influence the neutron distribution -> power reactor noise.
- These processes can be diagnosed by analysis of the induced neutron noise in a non-intrusive way during operation.
- This is achieved with a combination of core physics, advanced signal analysis and inverse methods.
- Swedish work has been performed in collaboration with the power plants and the safety authority.

The beginnings (Oak Ridge, 1969-70) Vibrations of a faulty control rod in the HFIR

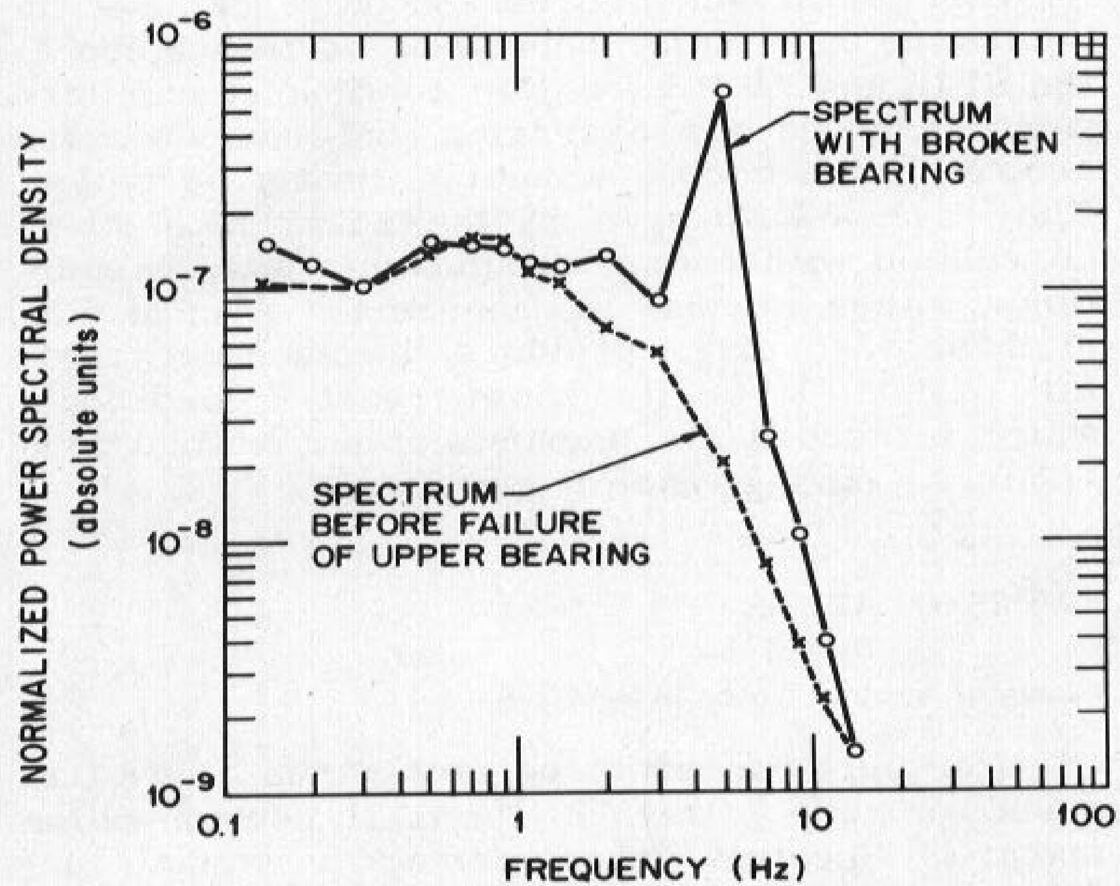
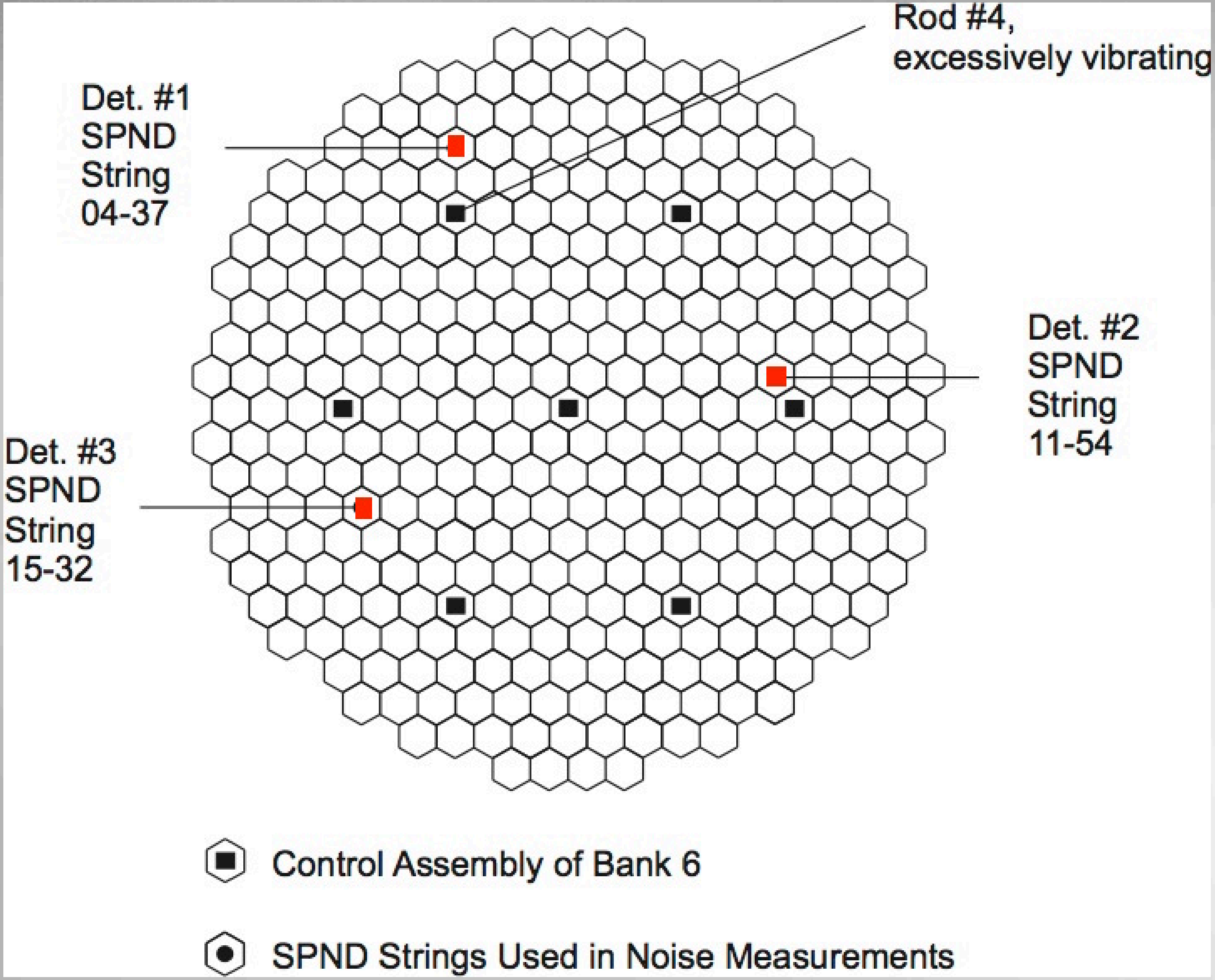
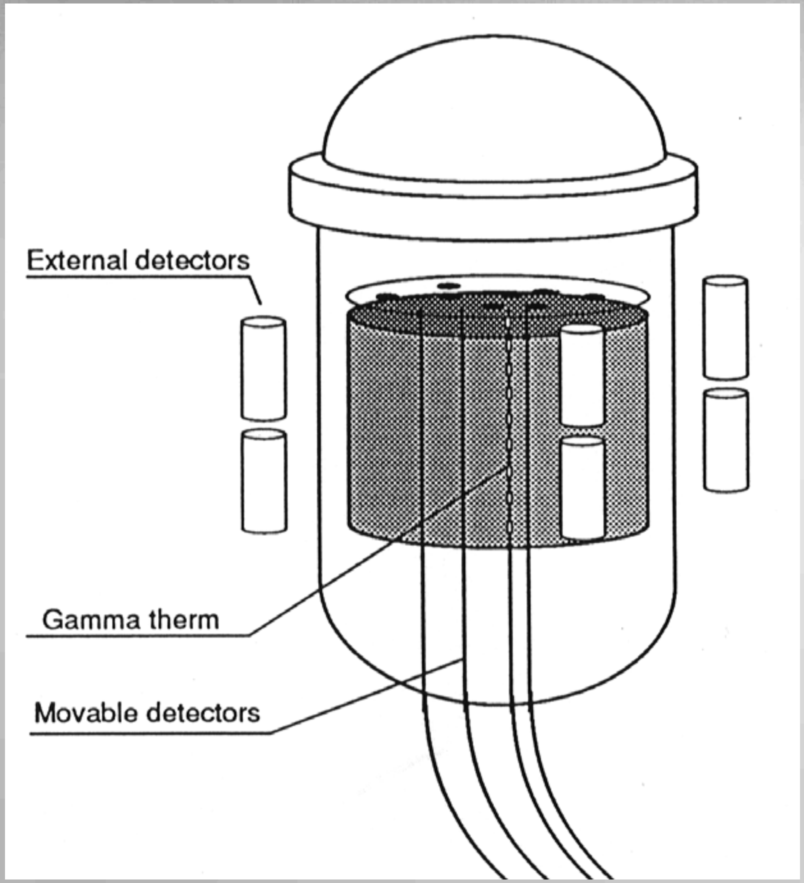


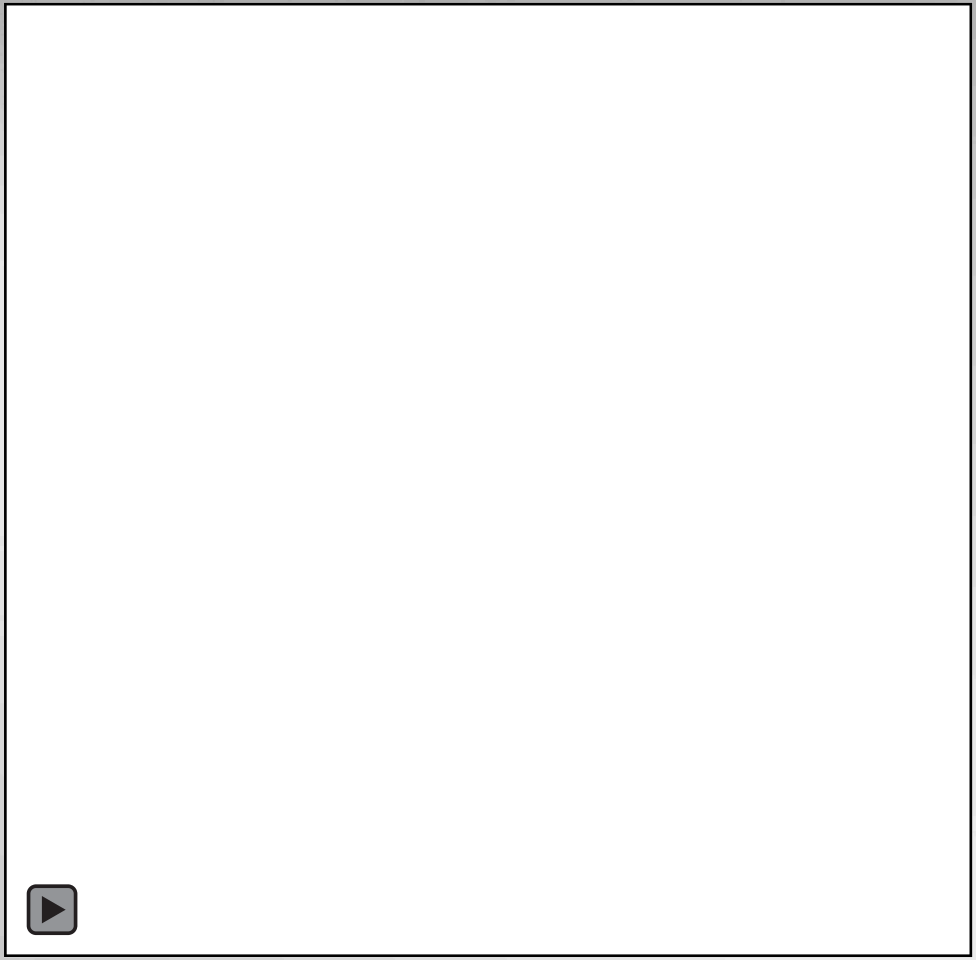
Fig. 4. Change in HFIR neutron-fluctuation spectrum as a result of rod bearing failure during fuel cycle 15.

Control rod vibations in the Paks-2 PWR, Hungary, 1986

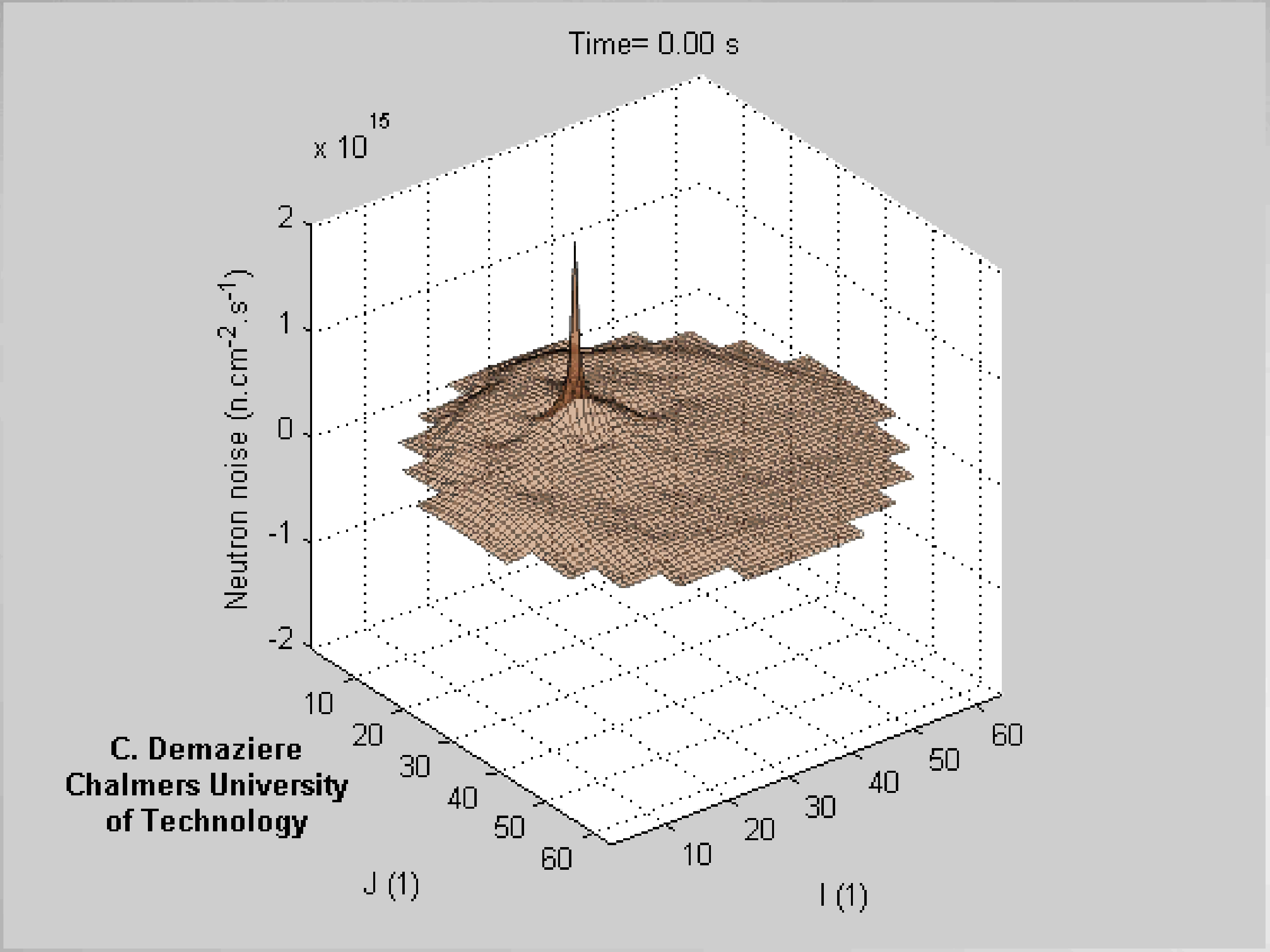


Core-barrel vibrations (Palisades, USA)

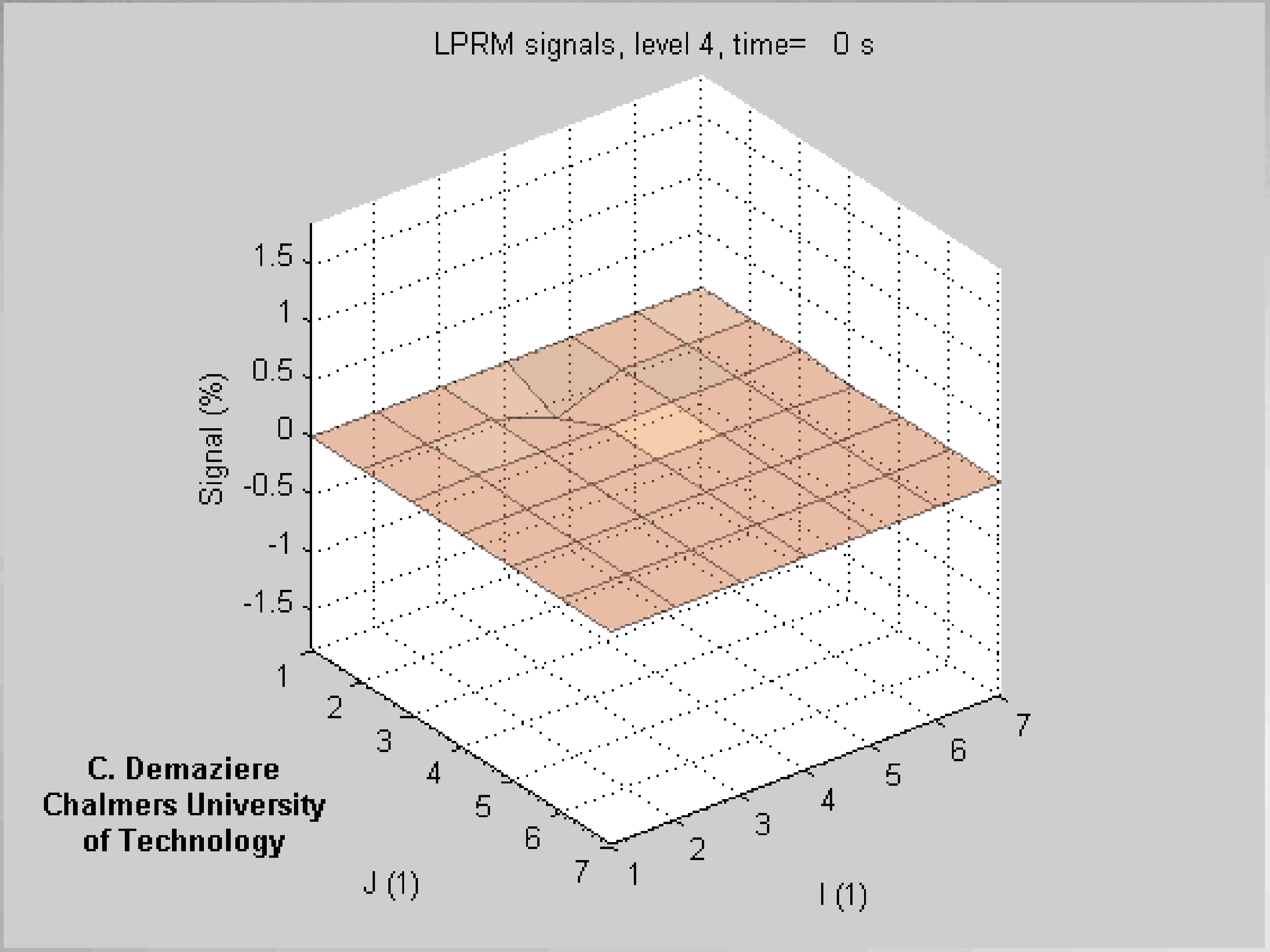




Swedish example:
Local BWR instability in the Forsmark 1 BWR, 1998



The Forsmark-1 measurement, 1998



Localisation of the channel-type instability

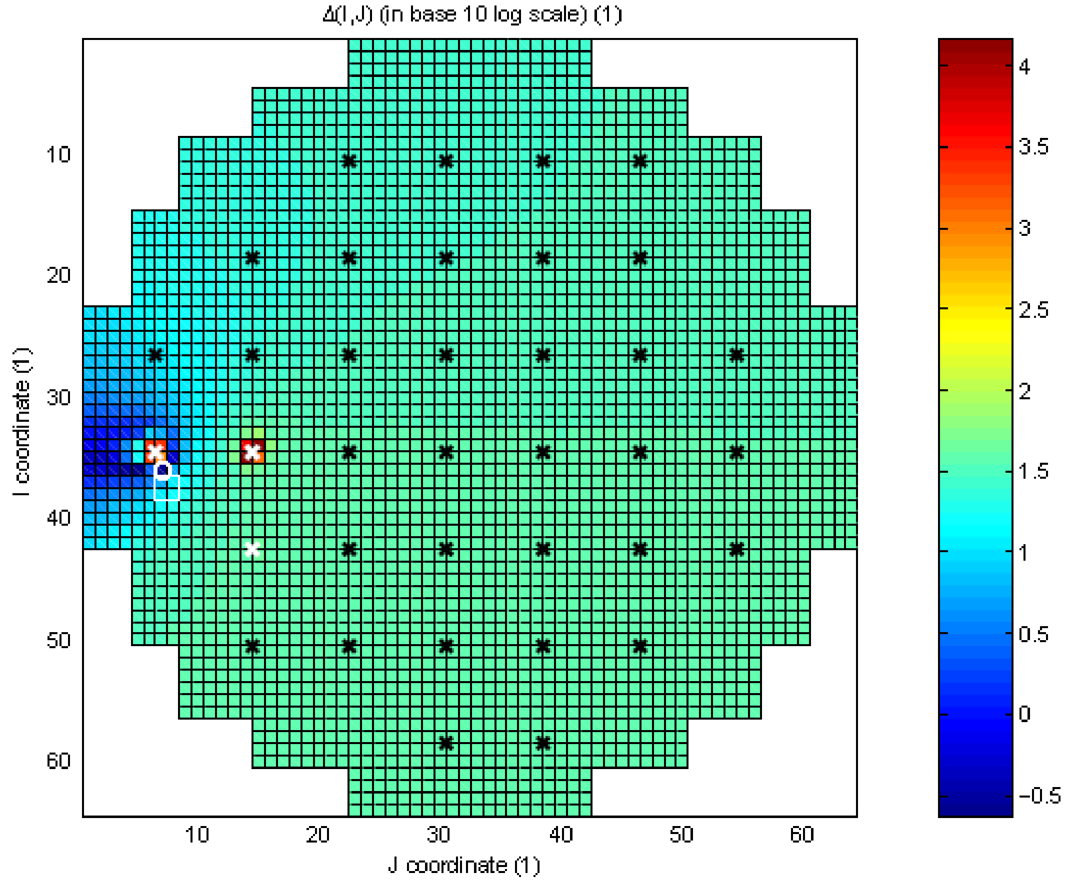
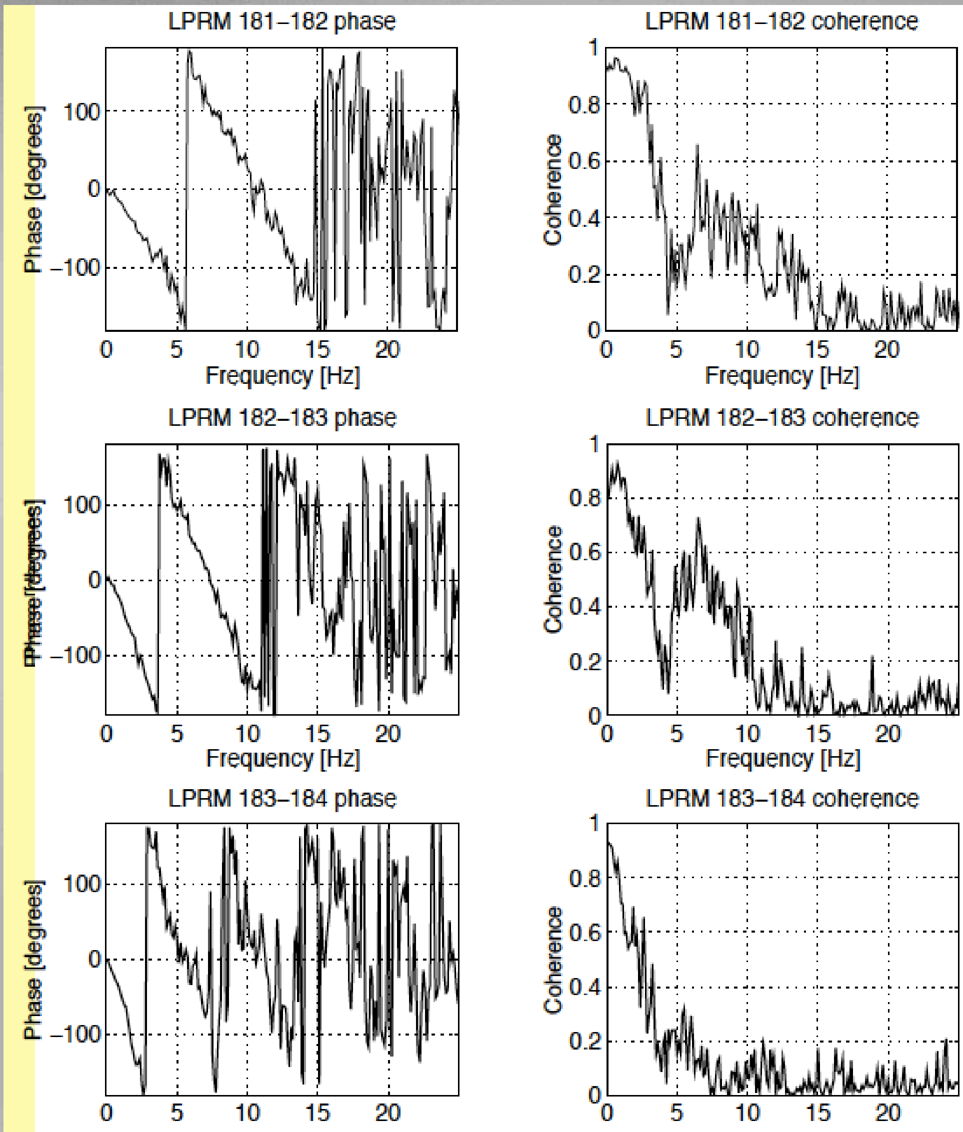
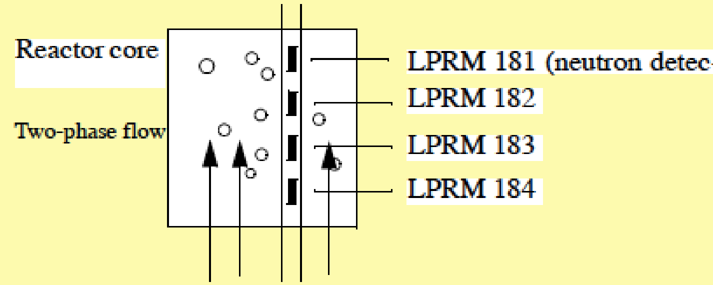


Fig. 2.14 Result of the localisation algorithm in the Forsmark-1 case (local instability event). The unseated fuel element is marked with a square, and the noise source identified by the localisation algorithm with a circle; the detectors that were used in the localisation search are marked by white crosses, whereas the detectors that were not used are marked by black crosses.

Coolant velocity measurements in a BWR (Barsebäck, Sweden – now phased out)





Normal behaviour

Coolant velocity distribution in the core from in-core neutron detectors (Paks-2 PWR)

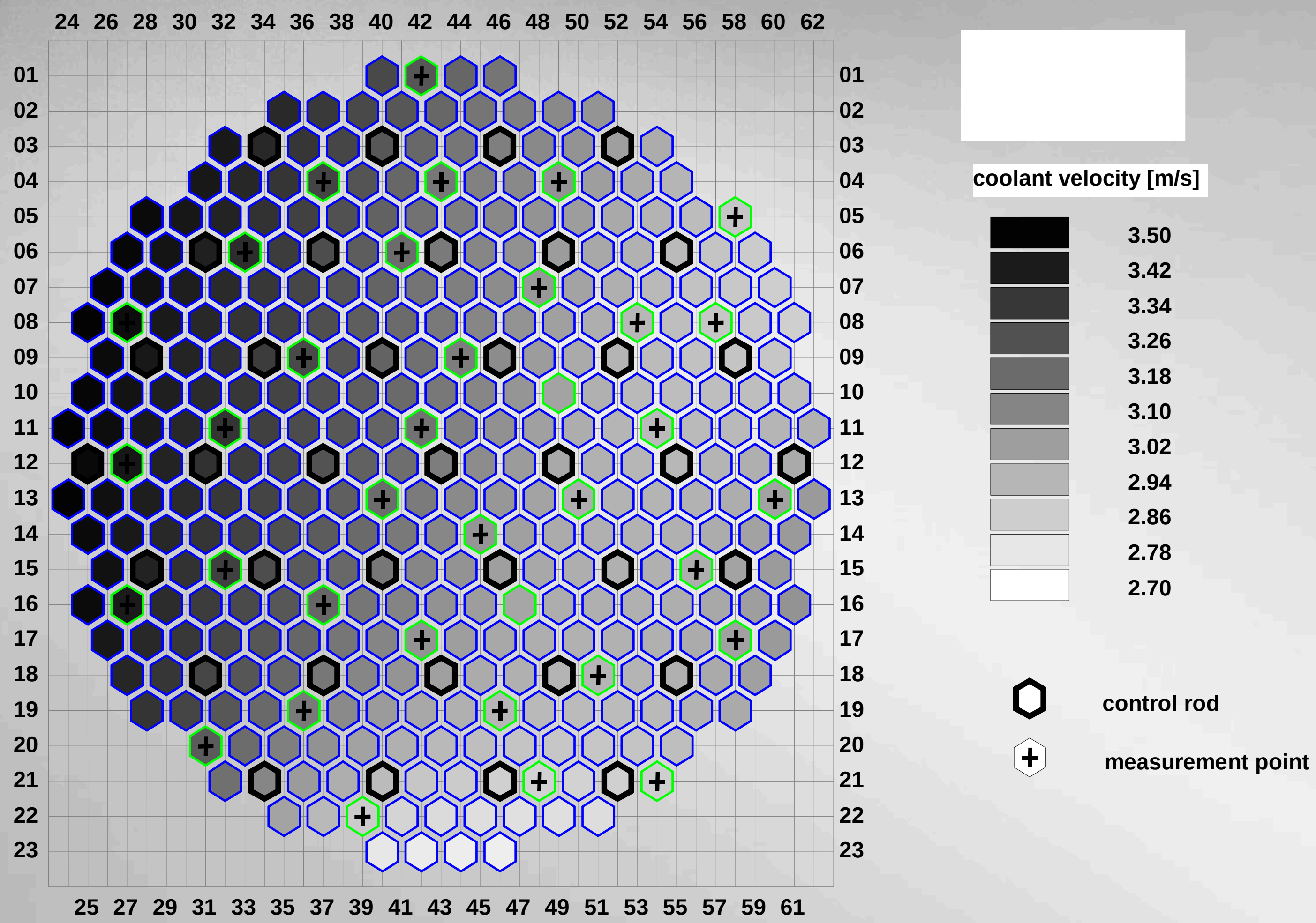


Illustration of the propagation of density perturbations in the core of an MSR

Neutron noise diagnostics: small time-dependent fluctuations

$$\frac{1}{v}\frac{\partial \phi(z,t)}{\partial t} = D\nabla^2\phi(z,t) + \left[\nu\Sigma_f(1-\beta) - \Sigma_a(z,t)\right]\phi(z,t) + \lambda C(z,t)$$

$$\frac{\partial C(z,t)}{\partial t} + u\frac{\partial C(z,t)}{\partial z} = \beta\nu\Sigma_f\phi(z,t) - \lambda C(z,t)$$

$$\Sigma_a(z,t) = \Sigma_a(z) + \delta\Sigma_a(z,t); \quad \phi(z,t) = \phi_0(z) + \delta\phi(z,t)$$

$$C(z,t) = C_0(z) + \delta C(z,t)$$

Linearization: neglecting $\delta\Sigma_a(z,t)\times\delta\phi(z,t)$

For details, see Pázsit and Demazière in the Nuclear Engineering Handbook, Edited by D. Cacuci, Vol. 3., and the Mathematica notebook, from the SAMOFAR web page.

Neutron noise (cont)

Substituting the splitting of the quantities, neglecting second order terms, after a Fourier transform one gets

$$\hat{L}(r,\omega)\delta\phi(r,\omega) = \delta\Sigma(r,\omega)\phi_0(r) \equiv S(r,\omega)$$

Solution with the Green's function technique:

$$\hat{L}(r,\omega)G(r,r',\omega) = \delta\left(r - r'\right)$$

$$\boxed{\delta\phi(r,\omega) = \int G(r,r',\omega)S(r',\omega)dr'}$$

Task: from the measured neutron noise $(\delta\phi(r,\omega))$, knowing the transfer function $G(r,r',\omega)$, to determine the perturbation $S(r,\omega)$.

Dynamic equation: traditional system

$$\nabla^2 \delta\phi(x,\omega) + B^2(\omega)\delta\phi(x,\omega) = \frac{\delta\Sigma_a(x,\omega)\phi_0(x)}{D} \equiv S(x,\omega)$$

where

$$B^2(\omega) = B_0^2\left(1 - \frac{1}{\rho_\infty G_0(\omega)}\right); \quad B_0 = \frac{\pi}{2a}$$

Solution: Green's function

$$\nabla^2 G(x,x_0,\omega) + B^2(\omega)G(x,x_0,\omega) = \delta(x-x_0)$$

$$\delta\phi(x,\omega) = \int_{-a}^{a} G(x,x_0,\omega)S(x_0,\omega)\,dx_0$$

Solution

$$G(x,x_0,\omega)=$$

$$-\frac{1}{B(\omega)\sin 2B(\omega)a}\begin{cases}\sin B(\omega)(a+x)\sin B(\omega)(a-x_0) & x\le x_0\\ \sin B(\omega)(a-x)\sin B(\omega)(a+x_0) & x>x_0\end{cases}$$

An illustration of the dependence of the Green's function on the frequency, system size and perturbation point is found in the Mathematica CDF file, downloadable from the Summer School web page.

Neutron noise in MSRs

$$\frac{1}{v}\frac{\partial\phi(z,t)}{\partial t} = D\nabla^2\phi(z,t) + \left[\nu\Sigma_f(1-\beta) - \Sigma_a(z,t)\right]\phi(z,t) + \lambda C(z,t)$$

$$\frac{\partial C(z,t)}{\partial t} + u\frac{\partial C(z,t)}{\partial z} = \beta\nu\Sigma_f\phi(z,t) - \lambda C(z,t)$$

$$\Sigma_a(z,t) = \Sigma_a(z) + \delta\Sigma_a(z,t)$$

$$\phi(z,t) = \phi_0(z) + \delta\phi(z,t)$$

$$C(z,t) = C_0(z) + \delta C(z,t)$$

The equation for the neutron noise (after linearisation and Fourier transform)

$$D\nabla^2\delta\phi(z,\omega)+\left[\nu\Sigma_f(1-\beta)-\Sigma_a-\frac{i\omega}{v}\right]\delta\phi(z,\omega)+\lambda e^{-\frac{\lambda(\omega)}{u}z}\frac{\beta\nu\Sigma_f}{u}$$

$$\left\{\frac{e^{-\lambda(\omega)\tau}}{1-e^{-\lambda(\omega)\tau}}\int_0^H e^{\frac{\lambda(\omega)}{u}z}\delta\phi(z,\omega)dz+\int_0^z e^{\frac{\lambda(\omega)}{u}z'}\delta\phi(z',\omega)dz'\right\}$$

$$=\delta\Sigma_a(z,\omega)\phi_0(z)\equiv S(z,\omega)$$

where

$$\lambda(\omega)=\lambda+i\omega$$

Simplifying the notations. Green's function

$$\nabla^2 G(z,z_0,\omega) + B^2(\omega)G(z,z_0,\omega) + \lambda e^{-\frac{\lambda(\omega)}{u}z}\frac{\beta\nu\Sigma_f}{Du}\times$$

$$\times\left\{\frac{e^{-\lambda(\omega)\tau}}{1-e^{-\lambda(\omega)\tau}}\int_0^H e^{\frac{\lambda(\omega)}{u}z} G(z,z_0,\omega)dz + \int_0^z e^{\frac{\lambda(\omega)}{u}z'} G(z',z_0,\omega)dz'\right\}$$

$$=\delta(z-z_0)$$

with

$$B^2(\omega) = B_0^2\left(1-\frac{i\omega\Lambda}{\rho_\infty-\beta}\right); \qquad \lambda(\omega)=\lambda+i\omega$$

Interpretation of the terms in the dynamic case

$$\nabla^2\delta\phi(z,\omega)+B^2(\omega)\delta\phi(z,\omega)+\lambda e^{-\frac{\lambda(\omega)}{u}z}\frac{\beta\nu\Sigma_f}{Du}\times$$

$$\times\left\{\frac{e^{-\lambda(\omega)\tau}}{1-e^{-\lambda(\omega)\tau}}\int_0^H e^{\frac{\lambda(\omega)}{u}z}\delta\phi(z,\omega)dz+\int_0^z e^{\frac{\lambda(\omega)}{u}z'}\delta\phi(z',\omega)dz'\right\}$$

$$=\delta\Sigma_a(z,\omega)\phi_0(z)\equiv S(z,\omega)$$

$$\delta C(z,\omega)=e^{-\frac{(\lambda+i\omega)}{u}z}\frac{\beta\nu\Sigma_f}{u}\left(\frac{e^{-(\lambda+i\omega)\tau}}{1-e^{-(\lambda+i\omega)\tau}}\int_0^H e^{\frac{(\lambda+i\omega)}{u}z'}\delta\phi(z',\omega)dz'\right.$$

$$\left.+\int_0^z e^{\frac{(\lambda+i\omega)}{u}z'}\delta\phi(z',\omega)dz'\right)\equiv\delta C_1(z,\omega)+\delta C_2(z,\omega)$$

The integral terms in the time domain

$$\delta C_2(z,\omega) = \frac{\beta\nu\Sigma_f}{u}\int_0^z e^{-\frac{i\omega}{u}(z-z')} e^{-\frac{\lambda}{u}(z-z')}\delta\phi(z',\omega)dz'$$

After inverse Fourier transform:

$$\delta C_2(z,t) = \frac{\beta\nu\Sigma_f}{u}\int_0^z e^{-\frac{\lambda}{u}(z-z')}\delta\phi(z',t-\frac{z-z'}{u})dz'$$

Similarly, for the first integral one obtains

$$\delta C_1(z,t) = \frac{\beta\nu\Sigma_f}{u}\sum_{n=1}^{\infty}\int_0^H e^{-\frac{\lambda}{u}(z-z')-n\lambda\tau}\delta\phi(z',t-\frac{z-z'}{u}-n\tau)dz'$$

Simplification to $u = \infty$

$$\nabla^2 G(x, x_0, \omega) + B^2(\omega) G(x, x_0, \omega) + \frac{\eta(\omega)}{T} \int_{-a}^{a} G(x, x_0, \omega) dx' = \delta(x - x_0)$$

with

$$B^2(\omega) = B_0^2 \left(1 - \frac{i\omega\Lambda}{\rho_\infty - \beta} \right); \quad B_0^2 < \frac{\pi}{2a}; \quad \eta(\omega) = \frac{\lambda}{\lambda + i\omega} \eta_0$$

Solution

$$G(x, x_0, \omega) = \frac{\eta(\omega)\phi_0(x,\omega)\phi_0(x_0,\omega)}{A^2 T K(\omega) B(\omega)^2 \cos B(\omega) a}$$

$$-\frac{1}{B(\omega)\sin 2B(\omega)a}\begin{cases} \sin B(\omega)(a - x_0)\sin B(\omega)(a + x) & x < x_0 \\ \sin B(\omega)(a + x_0)\sin B(\omega)(a - x) & x > x_0 \end{cases}$$

with

$$\phi_0(x,\omega) = A[\cos B(\omega)x - \cos B(\omega)a]$$

$$K(\omega) = B^2(\omega)\cos B(\omega)a + \frac{2a\eta(\omega)}{T}\cos B(\omega)a - \frac{2\eta(\omega)}{TB(\omega)}\sin B(\omega)a$$

Results. Classification of kinetic behaviour

The neutron noise is often split up into a reactivity or point kinetic term, and a space-dependent term:

$$\delta\phi(z,\omega) = \delta P(\omega)\phi_0(z) + \delta\psi(z,\omega)$$

$\delta\psi(z,\omega)$ is orthogonal to the static flux.

If the first term dominates, -> *point kinetic* behaviour

If the second component dominates, i.e. the space dependence of the noise deviates from the static flux -> *space dependent* behaviour.

5. Dynamic behaviour for u=∞: the Green's function

The point kinetic behaviour is retained up to higher frequencies (or system sizes) than in an equivalent traditional system.

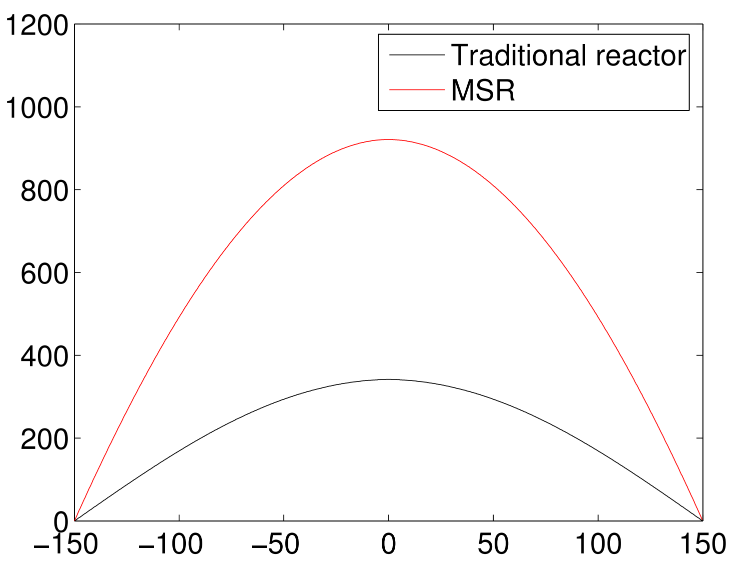


Figure: $\omega = 0.01$ rad/s

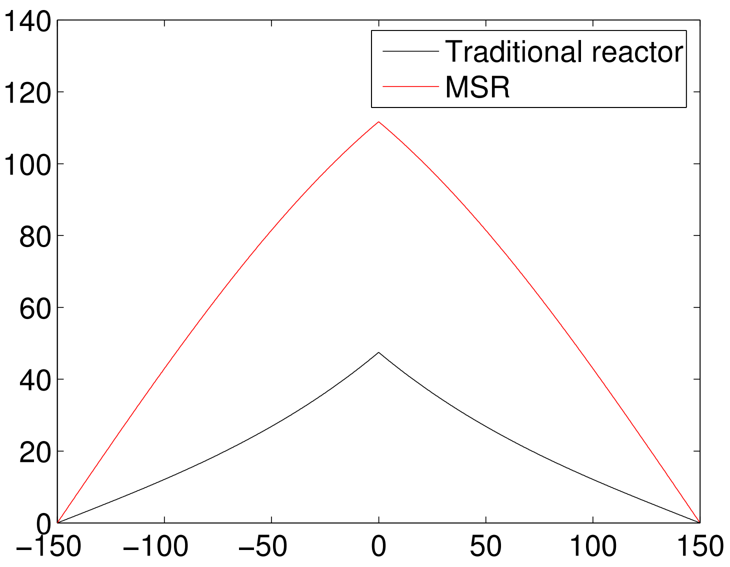


Figure: $\omega = 1$ rad/s

Dynamic behaviour for u=∞ (cont)

The physical reason is the spatial coupling, represented by the moving precursors and the smaller value of beta-eff.

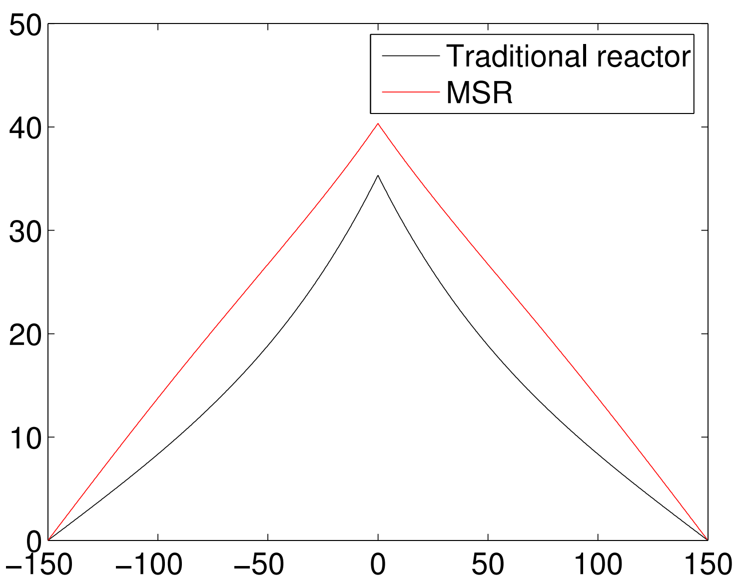


Figure: $\omega = 100$ rad/s

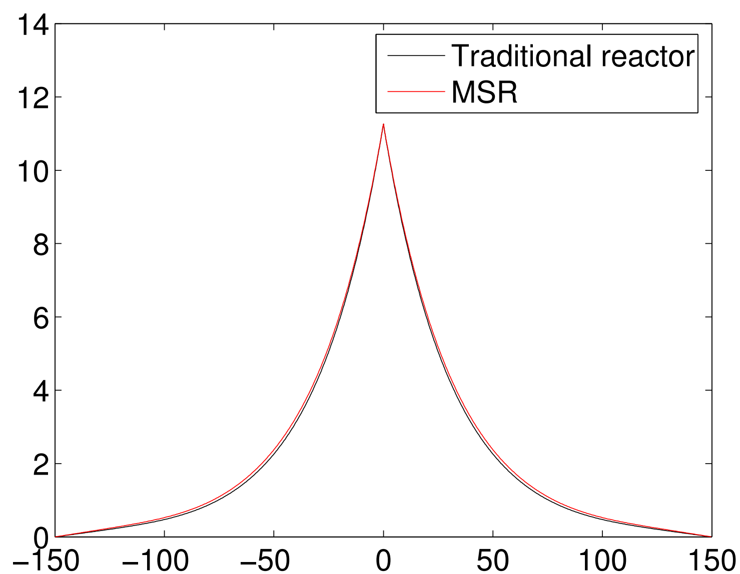
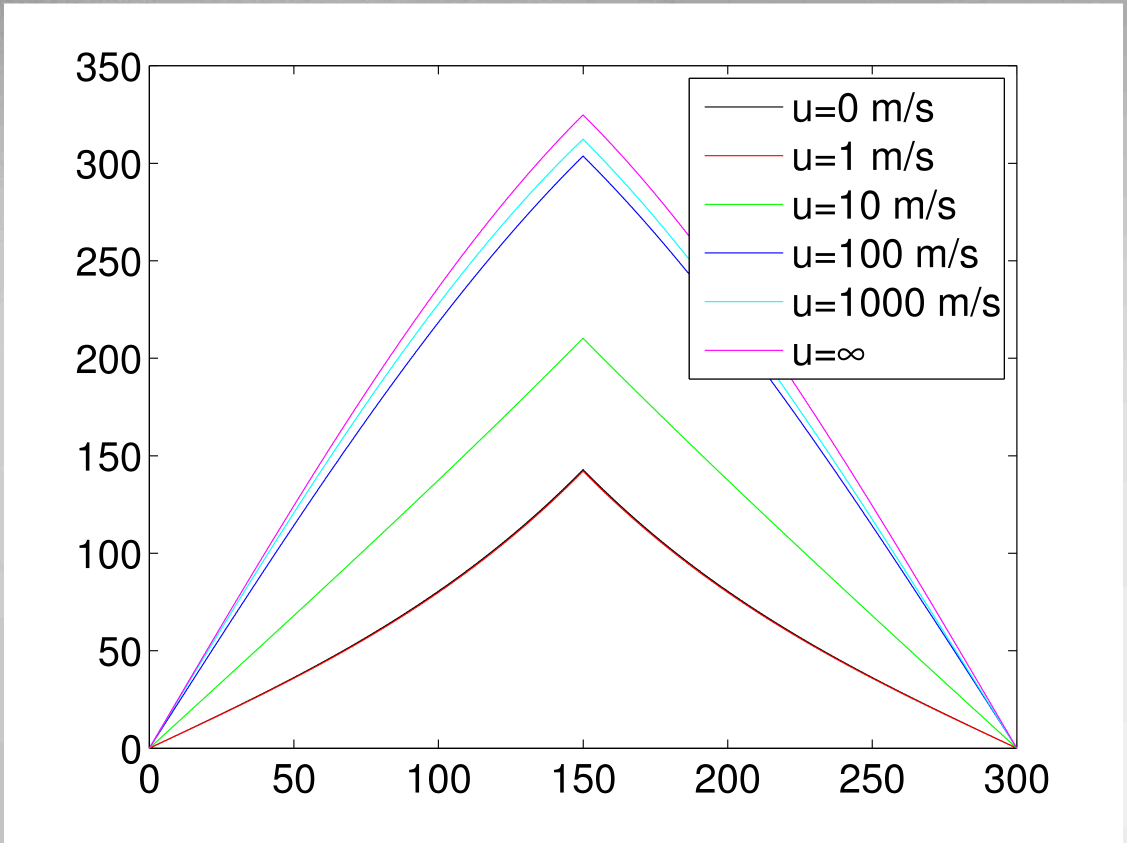


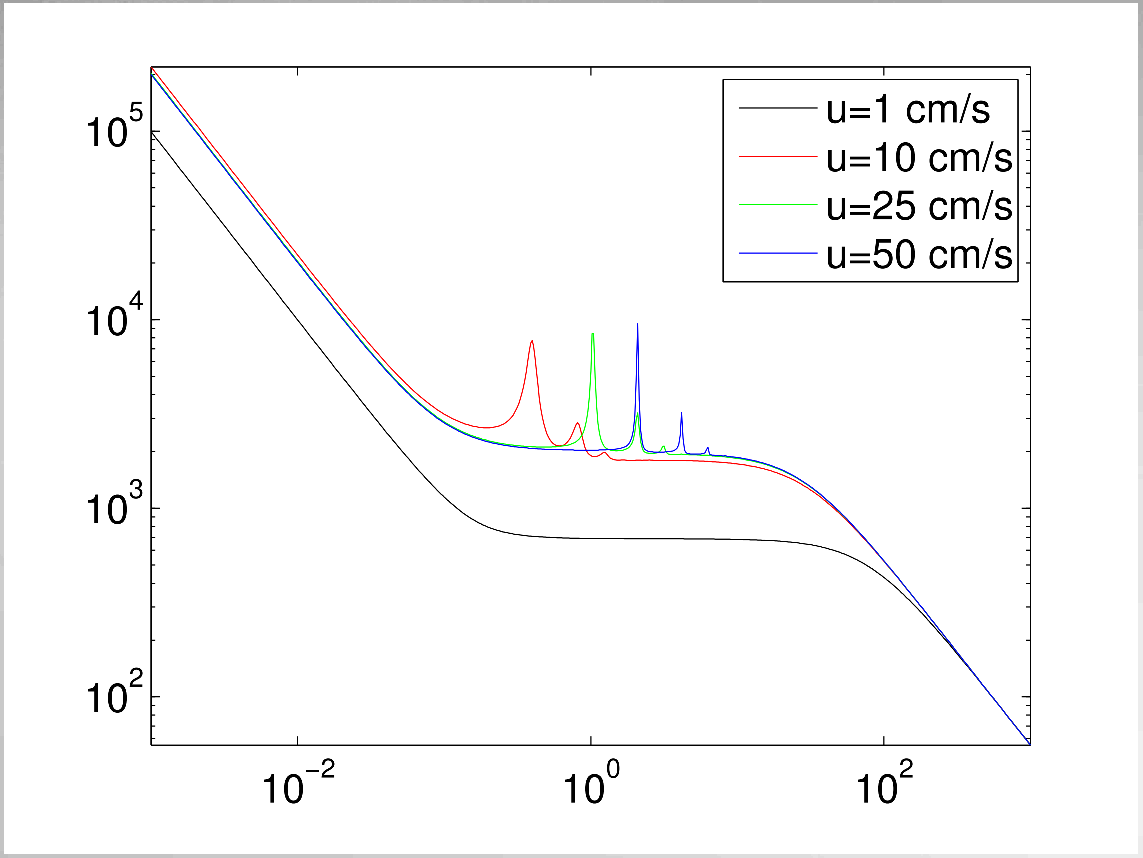
Figure: $\omega = 1000$ rad/s

Results for finite fuel velocity: space dependence



With the increase of the fuel velocity, the amplitude of the response increases, and its shape becomes more point kinetic. $\omega = 10$ rad/s.

Results for finite fuel velocity: frequency dependence



The frequencies of the ripples correspond to the multiples of the inverse of the recirculation time of the fuel (and hence that of the precursors)

The point kinetic approximation and the point kinetic component of the noise

Why is point kinetics and the calculation of the point kinetics interesting?

Because the relative contribution of the point kinetic component has a large influence on the possibility of recovering the noise source from the measured neutron noise.

For identifying the position of a localised perturbation, a strong point kinetic component is disadvantageous.

But its total absence, or a very localised transfer function is not optimal either.

Preliminaries and background

Traditional (solid fuel) reactors:

- The point kinetic equations can be derived by the Henry factorisation procedure;
- Together with the equation for the shape functions, the two coupled equations are equivalent to the starting diffusion or transport equation;
- Decoupling of the equations is achieved by the kinetic approximations, which make various assumptions on the shape function;
- In neutron noise theory, which is a linearized (first order) theory, the point kinetic approximation of calculating the amplitude function is “exact”, i.e. it gives the correct result in first order.

Preliminaries and background

Fluid fuel reactors (MSR):

- Derivation of the point kinetic equations is more involved (Ravetto, Dulla, Lapenta);
- The kinetic approximations do not decouple the equations for the amplitude and the shape function the same way as in traditional systems;
- In particular, when using linear neutron noise theory, application of the point kinetic approximations (using the static flux instead of the shape function), gives a result which is not correct in first order;
- The reason for this can be traced down to the fact that the definition of the adjoint for an MSR is different (i.e. “non-local”) from that in a traditional reactor.
- A “local” definition of the adjoint is not possible for MSR.

Preliminaries and background

However, the linearly correct form of the point kinetic component can still be calculated analytically, by an alternative way.

- This is because the full solution can be obtained analytically, and the point kinetic component can be obtained from it by projection.
- On the other hand, it is not possible to derive one single equation, which is not coupled to the shape function equation, and whose solution would yield the correct point kinetic term (= amplitude function).

Point kinetics: principles

Kinetic approximations: flux factorisation

$$\phi(z,t) = P(t)\psi(z,t)$$

together with the normalisation condition

$$\frac{\partial}{\partial t}\int_{-a}^{a}\phi_0^{\dagger}(z)\psi(z,t)dz = 0$$

P(t) is called the amplitude function, and $\psi(x,t)$ the amplitude function.

Point kinetics: principles

The normalisation condition can be written as

$$\int_{-a}^{a} \phi_0^\dagger(z)\psi(z,t)dz = \int_{-a}^{a} \phi_0^\dagger(z)\phi_0(z)dz$$

With this, one can recover the amplitude function (hence the point kinetic component) from the full space-time dependent solution as

$$P(t) = \frac{\int_{-a}^{a} \phi_0^\dagger(z)\phi(z,t)dz}{\int_{-a}^{a} \phi_0^\dagger(z)\phi_0(z)dz}$$

P(t) is usually derived from the point kinetic equations, which are generated from the full space-time dependent equations

Derivation of the point kinetic equations

Tools: the time-dependent diffusion equations, and the static equations for the flux and the adjoint.

$$\frac{1}{v}\frac{\partial\phi(z,t)}{\partial t} = D\nabla^2\phi(z,t) + \left[\nu\Sigma_f(1-\beta) - \Sigma_a(z,t)\right]\phi(z,t) + \lambda C(z,t)$$

$$\frac{\partial C(z,t)}{\partial t} + u\frac{\partial C(z,t)}{\partial z} = \beta\nu\Sigma_f\phi(z,t) - \lambda C(z,t)$$

$$\Sigma_a(z,t) = \Sigma_a + \delta\Sigma_a(z,t)$$

Point kinetic equations

One needs to factorise both the flux and the delayed neutron precursors

$$\phi(x,t) = P(t)\psi(x,t)$$

$$C(x,t) = C(t)\varphi(x,t)$$

with

$$\frac{\partial}{\partial t}\int_{-a}^{a}\phi_0^{\dagger}(x)\psi(x,t)dx = 0$$

and

$$\frac{\partial}{\partial t}\int_{-a}^{a}C_0^{\dagger}(x)\varphi(x,t)dx = 0$$

Point kinetic equations - derivation

The flux and precursor factorisations are substituted into the time dependent equations;

The time dependent flux and precursor equations are multiplied by the static adjoints of the flux and the precursors, respectively and integrated over the core;

The static adjoint flux and precursor equations are multiplied by $\psi(x,t)$ and $\varphi(x,t)$, respectively, and integrated over the core;

The latter set of equations for the amplitudes P(t) and C(t) is subtracted from the first, arriving at the point kinetic equations.

Point kinetic equations for an MSR

$$\frac{d}{dt}P(t) = \frac{\rho(t) - \bar{\beta}(t)}{\Lambda}P(t) + \bar{\lambda}(t)C(t)$$

$$\frac{d}{dt}C(t) + uS(t) = \frac{\bar{\beta}(t) + \bar{\rho}(t)}{\Lambda}P(t) - \bar{\lambda}(t)C(t)$$

Differences as compared to a traditional reactor:

- different definitions (weighting) of the parameters;

-the appearance of an extra term S(t) (adjointness);

- appearance of an extra reactivity term $\bar{\rho}(t)$

The non-adjoint property

Origin of the term S(t):

$$\int_{-a}^{a} C_0^\dagger(x)\frac{\partial}{\partial x}C(x,t)dx = \left[C_0^\dagger(x)C(x,t)\right]_{-a}^{a} - \int_{-a}^{a} C(x,t)\frac{\partial}{\partial x}C_0^\dagger(x)dx$$

$$\left[C_0^\dagger(x)C(x,t)\right]_{-a}^{a} = C_0^\dagger(a)\left[C(a,t) - C(a,t-\tau_L)\right] \neq 0.$$

Hence

$$S(t) \propto \left[C\dagger(x)\varphi(x,t)\right]_{-a}^{a} = C_0^\dagger(a)\left[\varphi(a,t) - \varphi(a,t-\tau_L)\frac{C(t-\tau_L)}{C(t)}\right] \neq 0$$

The solvable point kinetic equations

Neglect the term S(t) and assume the factoriations

$$\phi(x,t) = P(t)\phi_0(x)$$

$$C(x,t) = C(t)C_0(x).$$

Split up the amplitudes into expectations and fluctuations as

$$P(t) = P_0 + \delta P(t)$$

$$C(t) = C_0 + \delta C(t)$$

Then, after linearisation one obtains in the frequency domain

The solvable point kinetic equations

$$\delta P(\omega) = G_0(\omega)(\rho^{s}(\omega) + \rho^{sl}(\omega))$$

with

$$G_0(\omega) = \frac{1}{I\omega\left(\Lambda + \dfrac{\overline{\beta}}{I\omega + \overline{\overline{\lambda}}}\right)}$$

and

$$\rho^{sl}(\omega) = \rho^{s}(\omega)\frac{\lambda\beta}{I\omega + \overline{\lambda}}$$

However, this solution does not reconstruct the behaviour of the exaxt solution. It behaves just as smooth in frequency as that of a traditional reactor.

Empirical corrections

Empirical changes were suggested in the literature for the point kinetic equations, accounting for some delay effects:

$$\frac{dP(t)}{dt} = \frac{\rho(t) + \rho_f - \beta}{\Lambda} P(t) + \lambda C(t)$$

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda C(t) - \frac{C(t)}{\tau_c} + \frac{C(t-\tau_l)e^{-\lambda \tau_l}}{\tau_c}$$

Solution

One has

$$\delta P(\omega) = \rho(\omega) G_0(\omega)$$

with a modified zero power transfer function

$$G_0(\omega) = \frac{1}{i\omega\Lambda - \rho_f + \beta - \dfrac{\lambda\beta\tau_c}{1 + \tau_c(i\omega + \lambda) - \exp\{-\tau_l(\lambda + i\omega)\}}}$$

$$\rho_f = \frac{\beta(1 - \exp\{-\lambda\tau_l\})}{\lambda\tau_c + (1 - \exp\{-\lambda\tau_l\})}$$

However, this form does not reconstruct the exact solution either.

Exact solution

Can be determined from the full solution via the normalisation condition. Define

$$\delta\phi(x,\omega) = \delta P(\omega)\phi_0(x) + \delta\psi(x,\omega)$$

$$\delta C(x,\omega) = \delta C(\omega)C_0(x) + \delta\varphi(x,\omega)$$

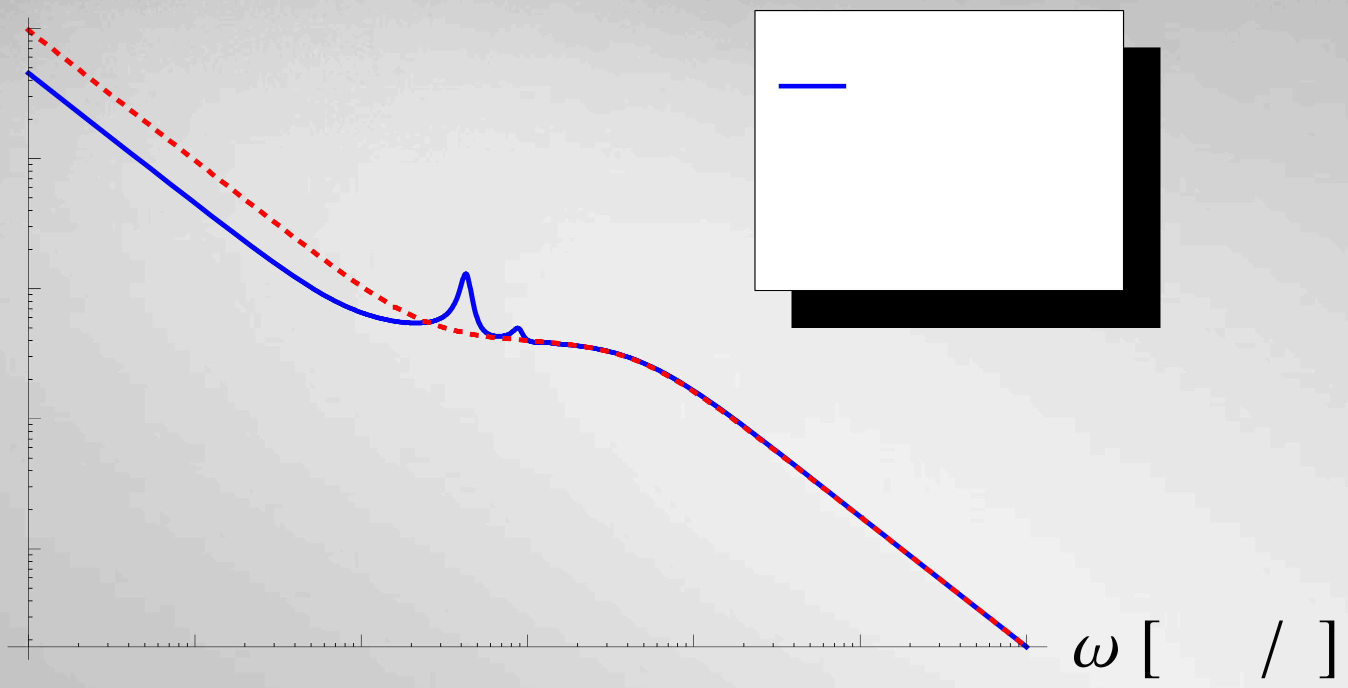
Then, due to the normalisation condition, $\delta\psi(x,\omega)$ will be orthogonal to $\phi_0^\dagger(x)$

Hence

$$\delta P(\omega) = \frac{\int_{-a}^{a}\phi_0^\dagger(x)\delta\phi(x,\omega)dx}{\int_{-a}^{a}\phi_0^\dagger(x)\phi_0(x)dx}$$

Comparison

$=$



Comparison between the solution of the point kinetic equations (red) and the exact solution (blue)

Corollary

Since the space-frequency dependent neutron noise (or its Green's function) can be calculated analytically in the present model, the point reactor component (the amplitude factor) of the noise can also be determined analytically.

However, one <u>cannot derive</u> point kinetic equations from the space-time dependent diffusion equations, whose solution is equal to the exact one.

7. Neutron noise in an MSR, induced by a perturbation propagating with the fuel

- Assume a disturbance (temperature/density fluctuations, inhomogeneous fuel distribution) which enters the core and propagates upwards unchanged in the fuel channel:

$$\delta\Sigma_a(z,t) = \delta\Sigma_a\left(0, t - \frac{z}{u}\right)$$

Neutron noise induced by a perturbation propagating with the fuel

- In the Fourier space:

$$\delta\Sigma_a\left(z,\omega\right) = \delta\Sigma_a\left(0,\omega\right)e^{-i\omega\frac{z}{u}}$$

$$S(z,\omega) = \delta\Sigma_a\left(0,\omega\right)e^{-i\frac{\omega}{u}z}\phi_0(z) =$$

$$const \cdot \phi_0(z)e^{-i\frac{\omega}{u}z}$$

The reactivity effect of the perturbation

$$\rho(t) = -\frac{1}{\nu\Sigma_f}\int_0^H \phi^2(z)\,\delta\Sigma_a(z,t)dz$$

$$\rho(\omega) = -\frac{1}{\nu\Sigma_f}\int_0^H \phi^2(z)e^{-\frac{i\omega}{u}z}dz$$

The APSD (auto power spectral density) of the perturbation

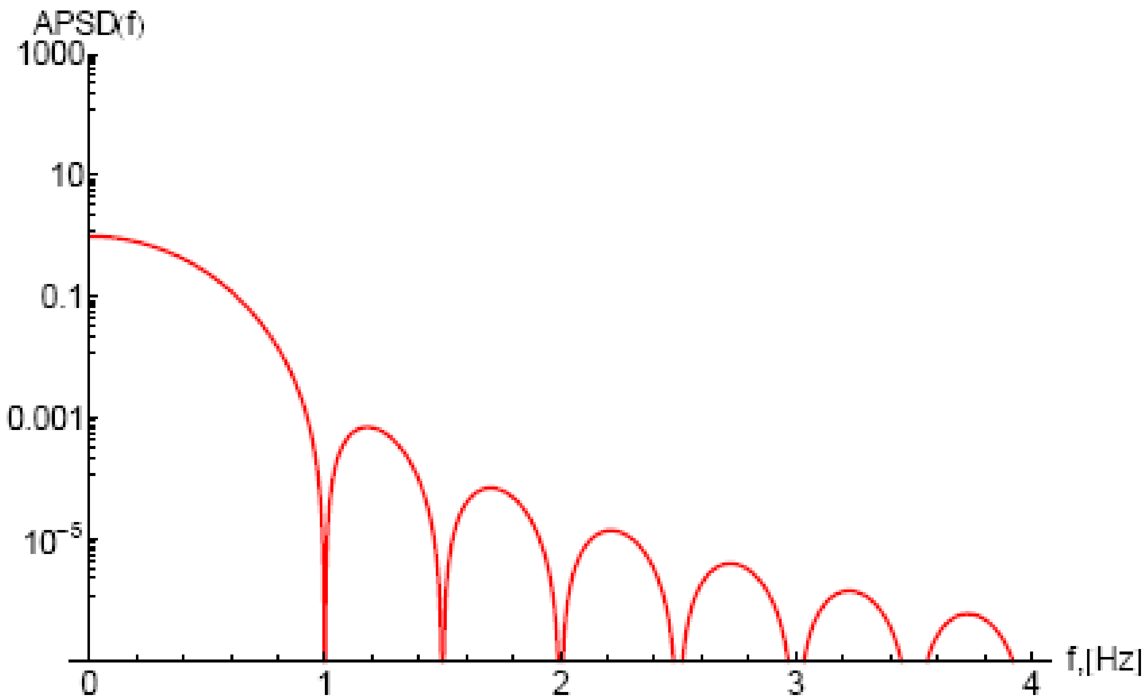


Fig. 1. APSD of the reactivity fluctuations due to propagating perturbations. T=2s, $f_T = 0.5$ Hz

The space dependence of the noise at a frequency where the reactivity effect is zero

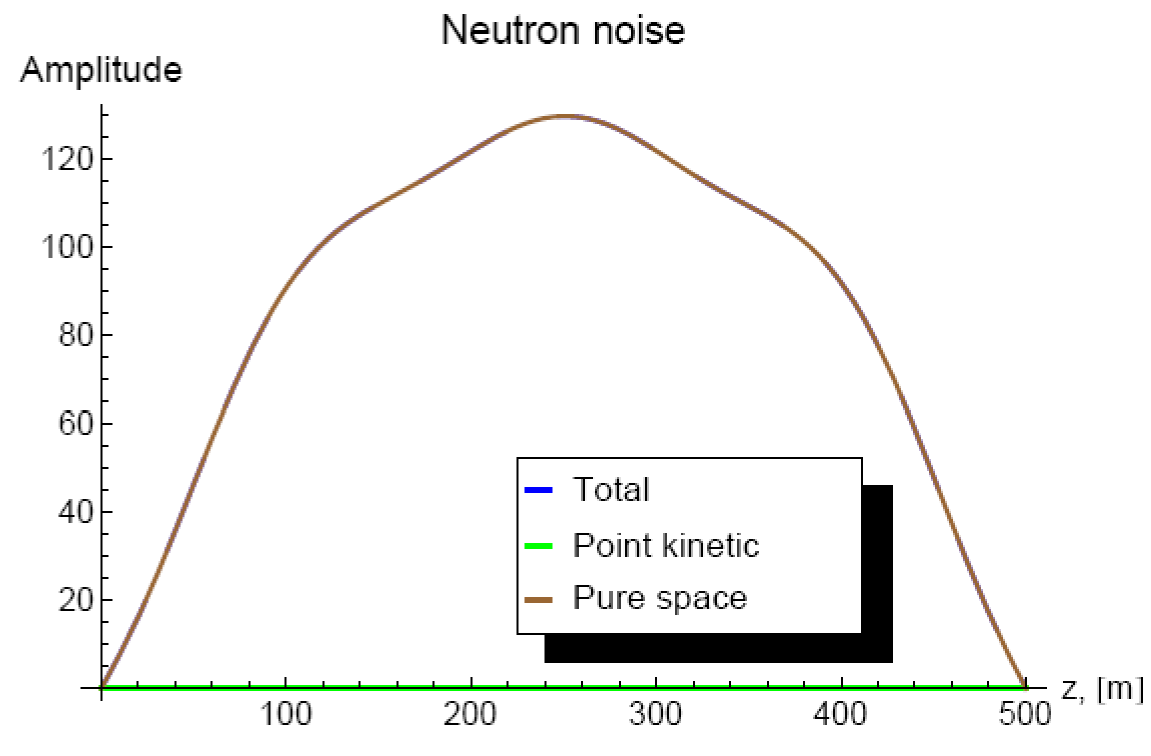


Figure 19: H=500cm,$\omega = 3\pi rad/s, v = 250cm/s$

The space dependence of the induced noise at three different frequencies

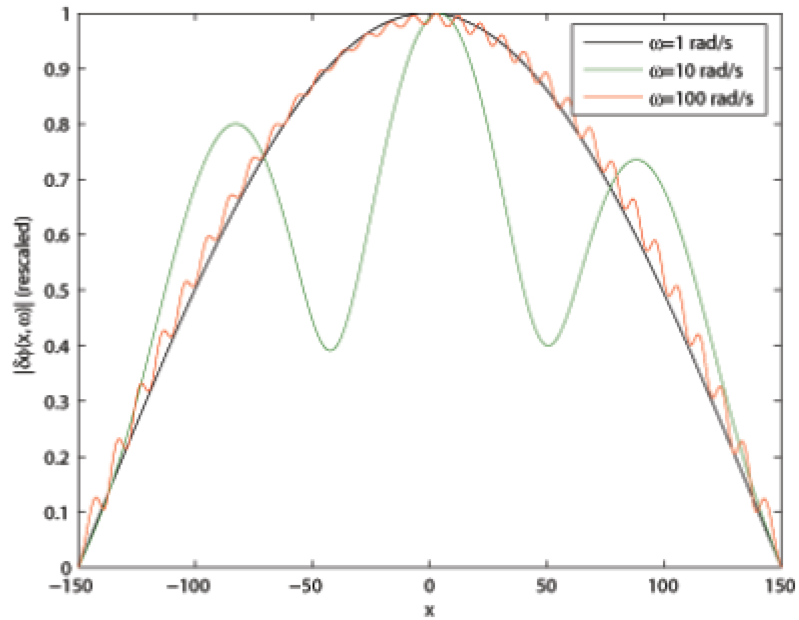


Figure: $\delta\phi(x, \omega)$ at different frequencies.

At intermediate frequencies, interference occurs between the point kinetic and space dependent components (green)

Significance

- The significance of the character of the interplay between point kinetic and space dependent components, as well as the frequency/system size domain where it is strong, is that it determines the possibilities of locating and quantifying a perturbation.

  Example: the Forsmark local instability event.

Conclusions

- The dynamic response of an MSR deviates in certain aspects quite markedly from that of traditional systems
- Hence the possibilities for diagnostics will be also different. In general, noise amplitudes will be higher and a more coupled (less space-dependent) response of the the core is envisaged
- In addition, new types of disturbances or phenomena can be expected, such as the increased significance of propagating perturbations. **New instrumentation may be necessary to fully exploit the possibilities for core surveillance and diagnostics.**
- Many intriguing new problems!